Phys 410
Spring 2013
Lecture #28 Summary
3 April, 2013

Up to this point we have considered Newtonian dynamics and Lagrangian dynamics. Now we consider Hamiltonian dynamics. The Lagrangian is written in terms of n generalized coordinates and their time derivatives. This set of parameters constitutes a $2n-$ dimensional **state space**. The Hamiltonian is written in terms of the generalized coordinates and their conjugate momenta, defined as $p_i = \partial\mathcal{L}/\partial\dot{q}_i$. This set of $2n$ parameters constitutes **phase space**.

One can solve the n canonical momentum equations for $\dot{q}_i$ in terms of the coordinates q_i and momenta p_i to arrive at $\dot{q}_i = \dot{q}_i(q_i, p_i)$. With this, one can express the Hamiltonian in terms of coordinates and momenta alone. Taking the derivative of the Hamiltonian with respect to q_i and p_i, one finds Hamilton's equations: $\dot{q}_i = \partial\mathcal{H}/\partial p_i$ and $\dot{p}_i = -\partial\mathcal{H}/\partial q_i$, $i = 1, \dots, n$. This is a set of $2n$ first-order differential equations, as opposed to the set of n second-order differential equations one gets from Lagrange's equations.

The Hamiltonian dynamics formulation is useful for quantum mechanics and for classical statistical mechanics. As a way of solving classical mechanics problems it has few advantages over Lagrangian dynamics.